A Synthesis Method of Electrostatic Gun Design

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This paper describes a method of electiostatic ion gun design in which the beam requirements are specified along a central trajectory The edge tiajectories which are internal ion paths of a thick beam consistent with this central tiajectory are then calculated Finally, the electrodes required to form the beam are theoretically determined Although previous investigators have used this approach with the paraxial ray equation, their results are not reliable for thick beams because the paraxial ray equation gives only an estimate of the behavioi of thick beams, and the eiror inci eases with the beam thickness Consequently a straightfoi ward extension of the paraxial method has been derived which is suitable for thick beams and can give beam and electrode shapes to any accuracy required Relationships have been obtained which indicate how to specify trajectory shape and potential to meet several desirable specifications, including the following: curved trajectory, reasonably uniform emitter cui1 ent density, specified accel-decel ratio, and nonintercepting exit electrodes parallel to the beam which can provide a saddle point suitable for electron injection A curvilineai hollow beam ion gun designed by this method is described

I Introduction

THERE are two alternative methods of designing ion guns which may be used either separately or in combination The first is an analysis method in which the electrodes are specified, and the beam that they produce is found either by numerical procedures or by analog techniques This technique involves several attempts and changes of the electrode shapes until a suitable beam is found The alternative technique, which will be discussed in this paper, is that of synthesis The properties of the beam, e g , its convergence, current density distribution at the emitter, and the exit conditions of the gun, are specified in detail, and the electrodes that will produce this beam are then determined The difficulty with this approach is that the electrodes so found may be awkward in shape or, worse still, completely impractical However, the intelligent choice of a beam can usually lead to electrodes that will not be physically impossible in shape A combination of the two techniques, synthesis and analysis, should allow the designer to obtain the best compromise for the beam and the electrode system This marriage of the two methods is the final aim; it should finally provide the ideal method of gun design

This paper will be divided into two parts The first part will describe how a paraxial type of gun with a curvilinear axis is designed With this approach the beam is assumed to be very thin and to have an arbitrarily shaped central trajectory The specified boundary conditions are the potential variation along the central trajectory, the uniformity of current density at the emitter, and the exit conditions for the beam From this information, by means of the paraxial equations of flow, the shape of the central trajectory is obtained, and, in addition, by the use of Harker's method,[1] the form of the electrodes is found This procedure has been used for the design of a hollow beam ion propulsion gun with the current density at the emitter of 10 ma/cm^2 Some of the results of this synthesis technique have been confirmed by a numerical analysis, adapted from the NASA Lewis program of Hamza,[2] working inward from the electrodes

The second part of the paper is devoted to a description of how the paraxial method can be extended to deal with beams of arbitrary thickness In this case, it is possible to find the variation of current density over the emitter, the shape of the emitter, and the beam trajectories far from the curvilinear axis The method has been tested successfully against known exact solutions, and an example is given of its application to extend our results on the paraxial ion gun design

II Design of a Paraxial Ion Gun

An ion propulsion gun should have emitter current density as high as emitter technology will allow, in order to reduce the inefficiency due to thermal energy radiated from the emitter Since the density of flow of neutral cesium through a porous tungsten plug should be uniform over the surface, the current density should be reasonably uniform over the face of the emitter The emitting surface should be large in comparison to the nonemitting hot surfaces associated with any practical emitter design Although the accelerating electrodes used to obtain this wide, dense beam may be at any reasonable potential, an accel-decel configuration is indicated, because with a high-potential accelerating electrode it is possible to obtain larger electrode spacings and hence a wider beam for a given variation in current density over the surface of the emitter The potential at the exit of the gun is prescribed by the overall requirements of the mission

In addition to these basic requirements, a curved beam has been chosen for the following reasons:

1) By avoiding a line-of-sight radiation path between the hot emitter and the outside environment, power loss by heat radiation is reduced

2) Beam curvature provides a converging force tending to counteract the diverging forces on the beam as it passes through the potential maximum near the accelerating electrode This allows greater flexibility in the choice of other design parameters In particular, a large accel-decel ratio can be used, and hence the beam length and width in the gun can be increased

3) There is the eventual possibility, with a strip beam, of using a two-sided emitter and thus increasing the thermal efficiency

The first step in the paraxial design method is to choose a central trajectory and the voltage profile along it The

Presented as Preprint 63043 at the AIAA Electric Propulsion Conference Colorado Springs, Colo , March 11–13, 1963; revision received November 1, 1963 The research reported in this paper was sponsored by the NASA Lewis Research Center, Cleveland, Ohio, under Contract NAS 8 1509

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paraxial ray equation is then used to calculate the thickness of this infinitesimally thick beam If this is satisfactory, the electrodes required to focus the beam are then calculated by the Harker method[1] to see if these possess a desirable shape

The next step is to assume that the beam has a small but finite thickness The boundary conditions at the beam edge are calculated by the use of a first-order theory, and the corresponding electrodes are determined Normally, these are very similar in shape to those in the infinitesimal beam

The final step is to calculate the boundary conditions at the edge of an arbitrarily thick beam based on the original central trajectory The corresponding electrodes are then calculated, and the design is complete

The designer would not ordinarily proceed immediately to the thick beam case, because it is the most complex and lengthy step and should, therefore, be undertaken only for the most promising designs Consequently, it is desirable to examine initially all designs using the paraxial approximations, in order to find the most promising cases and to eliminate those that are unprofitable

In the first step, the shape and potential profile of a central trajectory must be specified compatible with the requirements These will be defined in terms of s, the arc length along the emitter, as shown in Fig 1 It has been shown that, if the emitter is to be space-charge limited, a unique solution of the problem has then been determined [3] This treatment considers the design of a hollow beam gun rather than a strip beam gun, because it is believed to be mechanically simpler to construct and therefore easier to check the theory in detail, as was our purpose

The potential profile is chosen to meet several requirements simultaneously In the neighborhood of the emitter, the potential should vary as the four-thirds power of the distance It must be noted here that this method is valid only for space-charge-limited solutions Although an ion gun should not be operated in a completely space-charge-limited condition, the method will still provide a very close prediction of the beam shape under conditions in which the beam is barely temperature limited Accordingly, in the power series expansion of the potential in terms of s, the arc length along the central trajectory, the leading term should be $s^{4/3}$, and the potential should vary as $s^{4/3}$ near the emitter The potential then should rise to a maximum and subsequently should decrease to the exit value, with a suitable accel decel ratio

At the exit, it is required that the general region around the trajectory be at the exit potential and be essentially field-free If this can be accomplished, most of the lens effects of the exit aperture will have been designed into the gun itself, and the decel electrode will be nonintercepting At the exit, the beam will, in addition, flow as if it were in a unipotential drift tube, so that conditions will be suitable for neutralization To this end, it is required that, at the exit, both the first and second derivatives of the potential with respect to arc length must be small, and that the electric field normal to the beam axis must be zero The latter requirement is taken care of by specifying that the exit curvature of the central trajectory be zero It then follows from Laplace's equation that, if the second derivative of potential along the beam axis is zero, the second derivative of potential normal to the beam axis will also be zero Consequently, the exit region has been specified

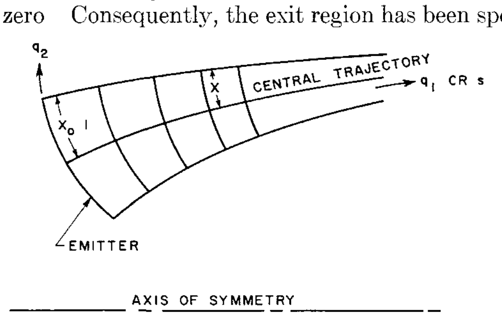

Fig 1 Coordinate system

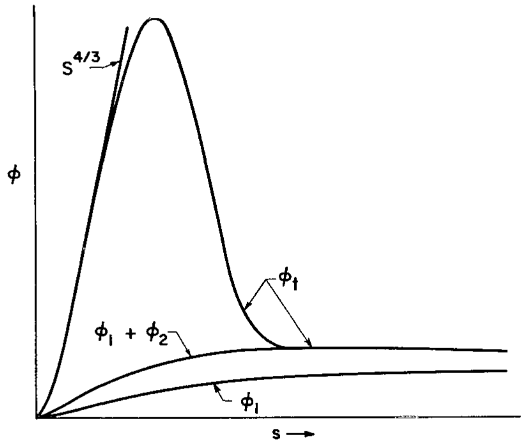

Fig 2 Potential profile

to be a saddle point, with the beam center and both exit electrodes at the same potential The effect of the space-charge of the beam has been ignored in these considerations, but the solution of the problem will include this effect It should be remembered, however, that the neutralization device at the exit will counteract the space-charge of the beam beyond the exit region

The best way of achieving the desired potential profile appears to be to make the potential the sum of two functions ϕ_A and ϕ_B, having the following characteristics: 1) The potential ϕ_A is chosen to vary as $s^{4/3}$ near the emitter and to increase monotonically to a fixed value at the exit, which is approximately at the potential of the exit electrodes 2) The potential ϕ_B is chosen to vary as $s^{4/3}$ near the emitter and to approach zero rapidly near the exit This term is mainly responsible for the rapid rise and fall of potential near the accelerating electrode Thus, by an appropriate choice of ϕ_A and ϕ_B, the conditions near the accelerating and decelerating electrodes can be controlled almost independently

Both of the potentials ϕ_A and ϕ_B are chosen to be entire functions, i e , to have no poles even for complex s except as $s \to \infty$ This serves to give rise to smooth electrodes Again, in order to obtain smooth electrodes, the functions $\phi_A(s)$ and $\phi_B(s)$ should be smoothly varying functions, and for large $s > s_{xit}$ we have the requirement that $\phi_A(s) \approx$ const and $\phi_B(s) \approx 0$ It is, therefore, possible only to represent these functions by infinite series; a polynomial would not satisfy the requirements

Two well-known entire functions that are suitable for this purpose are the exponential function and a Bessel function of imaginary argument which varies essentially exponentially for large s and as s^n for small s The component $\phi_A(s)$ is chosen to be of the form

$$\phi_A(s) = 0\ 1e^{-}\ [S^{1/2}I_{5/6}(s) + s^{4/3}]$$

$$= \phi_1(s) + \phi_2(s)$$

and $\phi_B(s)$ is chosen to be of the form

$$\phi_B(s) = 0\ 8s^{4/3}e^{P(\ )}$$

where

$$P(s) = 0\ 16s - s^4$$

In this representation, s is normalized and expressed in units of the emitter radius The total potential is

$$\phi_t(s) = \phi_A(s) + \phi_B(s)$$

and is plotted in Fig 2

It will be seen that $\phi_A(s)$ itself is the sum of two terms, $\phi_1(s)$ and $\phi_2(s)$ Because $I_{5/6}(s)$ varies as $s^{5/6}$ as $s \to 0$ and as $e\ /s^{1/2}$ for large s, $\phi_1(s)$ varies as $s^{4/3}$ as $s \to 0$ and becomes constant as $s \to \infty$ The second term varies as $s^{4/3}$ as $s \to 0$ and reaches

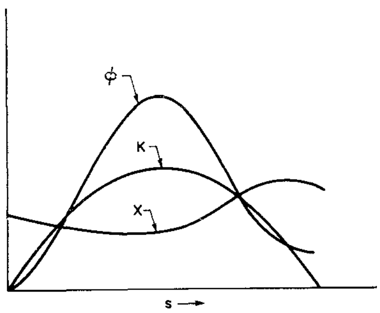

Fig 3 Curvature, potential, and infinitesimal beam thickness as a function of arc length

a very flat maximum at $s = 1\ 33$ The functions $\phi_1(s)$ and $\phi_2(s)$ are plotted in Fig 2 The function $\phi_A(s)$, then, is a function that levels off at a nearly constant value at the exit which should be in the neighborhood of $s = 1\ 6$

The form chosen for $\phi_B(s)$ is such that near the emitter $\phi_B(s)$ varies as $0\ 8\ s^{4/3}(1 + 0\ 16s)$, and this second term has the effect of supplying initial convergence to the beam, one of the means used to reduce the total beam divergence due to space-charge A potential of this form in the strip beam case indicates a converging cylindrical emitter In the axially symmetric case, it is necessary to examine the power series solution for the flow near the emitter [3] It is found that

$$\frac{dx}{ds} = -\frac{dr}{ds} - \frac{15}{8}\frac{d}{ds}(\phi s^{-4/3})$$

where x is the distance perpendicular to the beam axis between neighboring trajectories, normalized to unity at the emitter, and r is the distance to the axis of symmetry In turn, the power-series solutions for ϕ_A and ϕ_B must be examined in order to determine their contributions to convergence It is also necessary to assign the angle of the emitter with the axis, which in our case is 45° The contributions are then

$$dr/ds = +0\ 707 \text{ converging}$$

$$\tfrac{15}{8}\, d/d(\phi_A s^{-4/3}) = -0\ 375 \text{ diverging}$$

$$\tfrac{15}{8}\, d/ds(\phi_B s^{-4/3}) = +0\ 240 \text{ converging}$$

The value for dx/ds is then the sum of these terms, or $-0\ 572$, and its reciprocal is the radius of curvature of the emitter in the meridian plane, 1 75 The initial convergence is seen to be due to a combination of the initial slope and the choice of ϕ_B, overcompensating for the diverging effect due to ϕ_A

Until s becomes quite large, ϕ_B and hence ϕ_T vary essentially as $s^{4/3}$ This serves to keep the electrode spacings in the accelerating region comparable to those of the Pierce gun, rather than being unduly compressed on the side nearest the axis Figure 2 shows that the $s^{4/3}$ variation in ϕ_T is approximated a considerable distance from the beam axis For larger s, the s^4 term in the exponential becomes dominant and causes the $\phi_B(s)$ to approach zero rapidly as s is increased The ratio 1:1:8 of the three terms in ϕ_T is chosen in order to obtain an accel-decel ratio of about 6 to 1

In addition to the potential profile, the trajectory shape must also be specified The trajectory shape can best be described by giving the r and z coordinates of the point at which the trajectory originates, the initial angle of the trajectory with respect to the axis of symmetry, and the curvature of the trajectory as a function of arc length The position of a point for any s is then given by solving the system of ordinary differential equations:

$$dr/ds = \sin\psi$$

$$dz/ds = \cos\psi$$

$$d\psi/ds = K(s)$$

The initial angle is set at 45° more or less arbitrarily in order to provide enough total beam deflection angle to satisfy the requirements outlined earlier, while avoiding a difficulty that arose when designing a 90° gun, i e , for a 90° gun, the principle accelerating electrode comes very close to the inner beam-forming electrode, whereas for a 45° gun, the required spacing in this region is practically attainable Since the exit angle is necessarily zero (allowing the beam to leave parallel to the axis of symmetry), the integral of the curvature, $K = d\psi/ds$, over the trajectory must be $-\pi/4$

There is a relationship between the beam curvature at the emitter and the current density variation over the surface of the emitter This may be written as

$$(1/J)(dJ/du) = 5K$$

where u is the distance along the emitter surface For the current density to be uniform, it can be specified that $K = 0$ at $s = 0$, eliminating at least the first-order variations of current density with u At any point on the beam, the electric field perpendicular to the beam is $E = 2K\phi$ Since the exit region should be substantially field-free, this implies that the curvature at the exit must be zero or very small

The value of the curvature elsewhere is a compromise between two conflicting principles If the curvature is nearly zero everywhere but at the point of potential maximum, i e , the beam turns sharply through 45° near the potential maximum, the paraxial ray equation indicates that this would be the optimum way to limit the tendency of the beam to diverge On the other hand, the choice of an exponential function for $K(s)$ such as was used for the potential, although achieving this sharp peak, introduces regions of multivalued potential outside the beam This, in turn, gives rise to unreasonably complicated electrode structures To keep the electrodes smooth in shape and simple in form, $K(s)$ was chosen to be the simple parabola

$$K(s) = \text{const} \times (1\ 6s - s^2)$$

with the constant set to make the total beam deflection be 45° This function is plotted in Fig 3, where it is apparent that the potential maximum is near the point of maximum curvature

If the potentials in the exit region are again examined, it is seen that, although the normal field is zero at the exit, the derivative of the normal field with respect to arc length is finite This potential distribution corresponds to a saddle point with one equipotential parallel to the trajectory and one perpendicular to it

Figure 3 also shows the infinitesimal beam thickness for the specified boundary conditions, calculated from the paraxial ray equation The thickness of the beam, x, is given to first order by the paraxial ray equation

$$x'' = \frac{1}{2\phi}\left\{\frac{4}{9r\phi^{1/2}} - 4K^2x\phi - x\phi'' - x'\phi' - \frac{\phi'}{r}\, x\sin\psi - \frac{2k\phi}{r}\, x\cos\psi\right\}$$

in which the prime refers to differentiation with respect to arc length The first term on the right-hand side of the equation is the space-charge term It is particularly strong in regions of low potential and tends to make the beam diverge The second term on the right-hand side, involving the curvature K^2, represents a convergent effect tending to counteract the space-charge, as does the third term involving ϕ'' if it is positive In the region where the potential passes through a maximum near the accel electrode, the second derivative of potential is strongly negative, and so the ϕ'' term gives a strong diverging effect This is the reason for having the maxima of K and ϕ lie approximately at the same point in the trajectory With the final choice of the K and ϕ functions, the beam width never exceeds 1 45 times the starting value The plot in Fig 3 shows the beam as slightly convergent near the emitter (due to the potential specification), suffering a slight divergent force at the region of potential maximum, and reaching its greatest value of 1 45 near the exit

At this point the designer may proceed to step 2 and use first-order theory to calculate the electrode shapes for a beam of total thickness 0.15, being reasonably sure of the first-order approximation in this range. The boundary conditions along the central trajectory are now corrected to include a first-order approximation of the effects of space-charge, by adding to the normal field a term proportional to the emitter beam thickness and inversely proportional to r and the velocity.[4]

The electrodes used to produce this first-order approximation of the beam are then computed, using the method of Harker.[1] The resulting electrodes are shown in Fig. 4, along with the estimated beam thickness. For the dimensions shown in this figure, the emitter current and current density were calculated to be 42 ma and 10 ma/cm^2, respectively.

These approximate electrodes are now examined in detail to see if they are suitable for the gun. Several points may be noted concerning the electrodes:

1) The emitter surface is essentially a truncated cone, but, more accurately, there is a curvature in the meridian plane with a radius of curvature of approximately 1.75. Practically, the radius can be ignored, for with the specified emitter width of 0.15, the deviation from flatness is only 0.002.

2) The beam-forming electrodes are characterized by a pocket in the outer electrode which consists more accurately of two electrodes separated by a saddle point. Since the potential at the saddle is only 0.4% of the maximum beam potential, it has been replaced by a zero potential electrode surface.

3) The accel electrodes are at the same potential, but the inner one is closer to the beam, providing the necessary beam curvature. The spacing was estimated to exceed the expected deviation due to thermal velocities at the emitter.

4) The decel electrodes, which are at the same potential, are seen to flare out parallel to the beam at the exit, indicating that the fields are small in this region. This electrode configuration would appear to provide a reasonable match for a neutralization system, or electron trap, to follow the gun.

Although it is not the subject of this paper to discuss analysis methods that show for any electrode configuration the flows that can be expected to result from it,[2] we mention here the results of such an analysis, applied to the ion gun that we are discussing. The electrodes shown in Fig. 4, which are based only on first order approximations, were modified on the basis of practical experience to avoid excessive fields. The modified electrodes were then analyzed. The results of this analysis with the modified electrode shapes that were used are shown in Fig. 5. It can be seen that the beam shape is substantially the shape predicted by the first-order approximation, and the computed current is also close to the expected value.

It is postulated, however, that a more desirable beam can be formed if the electrodes from the complete thick-beam solu-

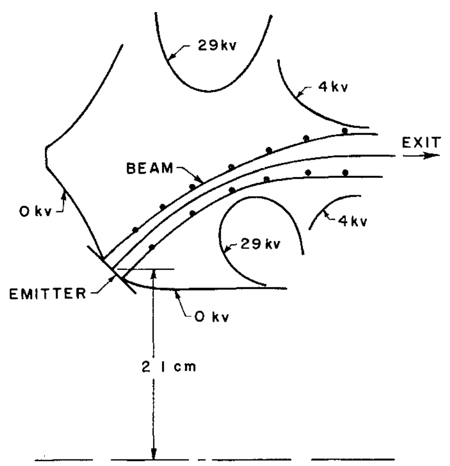

Fig. 4 Electrodes and flow: first-order approximation

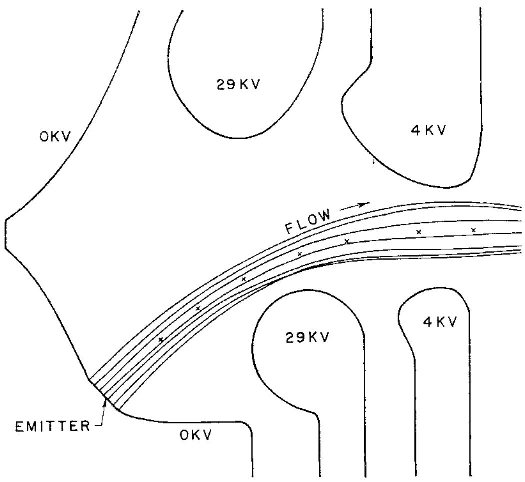

Fig. 5 Flow obtained by electrode analysis

tion are determined before modifications are attempted. All of the steps taken up to the present time have been based upon an exact specification of the central trajectory values but only on an approximation of the actual behavior of the beam. These steps represent preliminary checks on the proposed boundary conditions which can be carried out relatively easily to allow for the selection of the optimum set from a number of alternates. The solution of the thick-beam equations in detail, however, can describe the beam and the electrodes to as great a degree of accuracy as might be required. This procedure is developed in the following section.

III Complete Thick-Beam Solution

It is known that, for all regular flows, in the absence of magnetic field, the velocity can be expressed as the gradient of a scalar called the action potential. This forms the basis of our coordinate system, which is shown in Fig. 1. The variable q_1 is chosen to be some function of the actual potential, the emitter surface having the coordinate $q_1 = 0$, and the surfaces $q_1 =$ const being surfaces of constant action. The coordinate q_2 is chosen to be some measure of length along the emitter and serves to label the trajectories, each surface of $q_2 =$ const corresponding to a particular trajectory. It is convenient to define q_1 and q_2 in such a manner that on the central trajectory q_1 equals trajectory arc length s, and on the emitter q_2 equals emitter arc length u.

This coordinate system will be orthogonal, and the distance element can be described by the expression

$$dl^2 = h_1^2 dq_1^2 + h_2^2 dq_2^2 + h_3^2 dq_3^2$$

In this method all of the equations are to be expressed in terms of these generalized coordinates, and the values of h_1 and h_2 are to be determined at all points of interest. All of the information can then be mapped in the r,z meridian plane of an axially symmetric system, thereby solving the problem. In the axially symmetric case, q_3 is simply θ, and h_3 is r.

The relationships required by the ion flow are Poisson's equation,

$$\nabla^2\phi = -\rho/\epsilon$$

the equation of conservation of energy,

$$mv^2 = -2e\phi$$

and the equation of continuity;

$$\nabla \cdot (\rho v) = 0$$

In addition, certain geometrical relationships are required in order to make our coordinate system a real one. The combination of these equations yields a system of partial differential equations with q_1 and q_2 as the independent variables, and the four dependent variables h_1, h_2, r, and z It becomes apparent that the solution will map the trajectories in the r,z plane It can also be shown that this information is enough to give velocities, potential, fields, and space-charge—in short, everything needed to be known about the beam This is brought about as follows

Because the lines of const q_1 are lines of constant action, along these lines the velocity is inversely proportional to h_1, and, consequently, the potential is inversely proportional to h_1^2 If the potential along the central trajectory is prescribed as a function of arc length, or $\phi(q_1)$, then the potential at any point q_1,q_2 is simply $\phi(q_1)/h_1^2$ taken at the point in question By similar reasoning, the fields may be determined at any point in the domain of interest

The current density J along any trajectory is inversely proportional to rh_2 It can be obtained, therefore, from the product $J_0r_0h_{20}$, which will be shown to be a known function of q_2 The space-charge at any point is then the quotient of this current density and the velocity The equations reduce to nine first-order partial differential equations The exact form of these relationships is given in Ref 3

The supposition that the boundary conditions can be prescribed along the central trajectory requires certain precautions, since the trajectory is a characteristic of the equations Conditions along the central trajectory are completely known if the three quantities, potential, current density, and curvature, are given as functions of arc length However, the paraxial-ray equation, which is one of the equations in the set, determines any one of these quantities if the other two are specified To avoid overspecifying the boundary in this case, only the potential and the curvature will be defined

Because of the interrelationship along the trajectory, more boundary data must be given; resulting in a mixed boundary-value problem These remaining data can be given along the emitter surface and consist of the statement that voltage and fields are zero on the emitter, as required by space-charge-limited flow A similar solution could be found for the case in which the current density is not completely space-charge limited, and a different restriction would be placed on the emitter

With the boundary conditions along the center trajectory specified, one may examine first the solution of the equations in the vicinity of the emitter, where unfortunately the system cannot be solved by finite difference methods because one of the equations contains an indeterminate term However, in this region a power series solution has been obtained, with expressions giving the value of the coefficients through the third order in terms of the specified boundary conditions

For the power series, the potential along the central trajectory is reduced to the series

$$\phi = s^{4/3}(A + Bs^2 + Cs^3 + \quad)$$

This type of potential variation is sufficient to relate emitter current density to the metric h_1 Sufficiently close to the emitter, Child's law predominates, and, by defining a small increment q_1 and the corresponding potential on the central trajectory as ϕ_0, one obtains the relationship

$$J = \text{const} \times \frac{\phi_0^{3/2}h_1^{-3}}{\Delta q^2 h_1^2}$$

or

$$J = h_1^{-5}$$

where J is the current density on the emitter normalized with respect to the value for the central trajectory

The power series shows that a necessary and sufficient condition for the complete specification of a beam is the specification of potential and shape of the central trajectory The power series solution gives the emitter shape and the emitter current density distribution

Since the power series solution provides a set of values for the unknowns at a small distance away from the emitter, the necessary boundary values are available for numerical integration of the equations To integrate away from a trajectory $q_2 = q_{20}$, most of the nine unknowns are found for the new trajectory $q_2 = q_{20} + \Delta q_2$ Then, beginning at the power series solution, the remaining unknowns, including h_2, are found for the new trajectory, each set of values at $q_1 = q_{10}$ being used to solve for the point $q_1 = q_{10} + \Delta q_1$ The latter process may be thought of as the solution of a paraxial equation for a noncentral trajectory and is necessary because the trajectories are the characteristics of the partial differential equations involved

In the numerical solution of the partial differential equations, a difficulty arises Poisson's equation, which is one of the equations necessary in solving this problem, is elliptic and is unstable to computation by finite difference methods It is therefore necessary to use a technique of analytic continuation, which has been used in the past, to insure stability In this method, the independent variable q_1 becomes a complex number $q_1 + i\bar{q}_1$, thus introducing an additional independent variable $\bar{q}_1$ The boundary conditions are continued analytically with respect to this complex variable as well as to the equations themselves If this analytical continuation is made in a proper fashion, the result is a hyperbolic system that is stable to computation by finite differences The maximum step size for a stable solution can also be determined

It must be emphasized that the boundary conditions that have been chosen for the central trajectory, namely potential and curvature, must be analytic functions of arc length Although it is tempting to piece together two functions or to use empirical curves, such functions are not properly defined for complex values of arc length and are therefore excluded from the synthesis method For a physical explanation, it can often be demonstrated that piecewise continuous trajectories cannot be obtained from a system of electrodes that do not directly touch the trajectory

The analytic continuation that is necessary for the flow solution also serves another useful purpose The integration of Laplace's equation out from the beam edge also requires analytically continued boundary values Since these are available from the flow equations, all of the information required at the beam edge is available for finding the electrodes

As a test of the accuracy of the synthesis method, it was applied to the design of a gun for which the exact solution was already known This gun, shown in Fig 6, has a curvilinear flow inward from a hollow cylindrical cathode The exact solution for flow and electrodes was known, since the flow is in the class of two symmetries, all trajectories having the same shape By taking from the known solution only the ordinary differential equations describing potential and curvature along the central trajectory, and the initial slope of the trajectory with respect to the axis (directly downward), both the

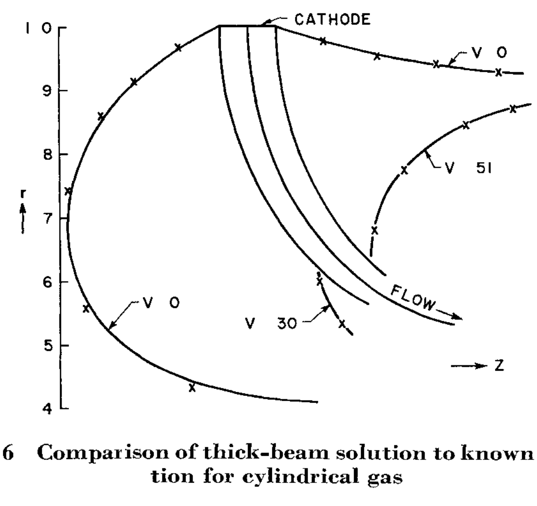

Fig 6 Comparison of thick-beam solution to known solution for cylindrical gas

boundary data for the central trajectory and the power series solution for the cathode region were developed, and the equations were applied to determine the flow and the electrodes The cathode shape was found from the power series and agreed with the known shape As shown in Fig 6, the difference between the two methods is practically insignificant for the accelerating electrodes and the trajectories, the main deviation being in the location of the beam-forming electrodes Presumably these would be reduced by using a smaller mesh size The mesh size along the trajectory was 0 02 times the cathode radius, and the flow equations were integrated five steps away on both sides of the central trajectory The whole design took 19 min on an IBM 7090 computer

The thick-beam solution may now be used to obtain accurate electrodes for the ion gun Using the central trajectory previously chosen as the boundary data, the power series solution is first found, describing the emitter and the flow in the emitter neighborhood The emitter shape is slightly deformed from the arc of a circle described previously The refinement, a distance of about 0 001, is not large enough to be of practical importance On the other hand, the power series solution gives important information about the current density distribution along the emitter The first-order variation is zero, as specified, and the current density is of the form

$$J = (1 - 0\ 0452q_2^2 + 2\ 96q_2^3)^{-5}$$

It will be seen that, for the chosen value of beam thickness, the extremes of current density occur at the edges of the beam, where $q_2 = \pm 0\ 075$, with the current density at the lower edge of the beam being about 1 5% larger than the minimum occurring at the upper edge In this range the deviation varies roughly as the cube of the thickness On the other hand, if the trajectories had been arcs of circles, with constant curvature even at the emitter, the current density variation would have been about 40%, increasing linearly with thickness The specification of zero curvature at the emitter is therefore very useful

Starting from the power series solution, the equations of flow are integrated for the whole beam, taking about 12 min on an IBM 7090 computer The electrodes are then determined and are found to be substantially unchanged from the electrodes shown in Fig 4, except that the left beam-forming electrode is moved in from the saddle point region The fields are low in this region, so that a displacement of an electrode here has small effect on the potentials elsewhere; consequently, this error in the first-order solution is relatively unimportant

In Fig 4, the dots indicate the edge trajectories determined by integrating the beam equations The integration places the edge trajectories above their predicted location at the exit It does not, however, move the exit electrodes a corresponding amount With the diminished spacing, it can be expected that the top exit electrode will intercept too many of the ions with high thermal velocities The remedy is simply to choose a slightly lower potential for the exit electrodes Their location will then be found to be farther away from the beam, without the beam solution being altered

It should also be pointed out that the amount of interception to be expected at the exit electrodes will be reduced in practice when the gun is being operated at a current that is slightly less than the space-charge-limited value This is shown by the analysis that was done on the gun of Fig 5, in which the exit convergence is greater when the space-charge is reduced

The aim of this study was to develop a precise method for the computer design of an ion propulsion gun in order to aim for very low interception rates Several of the requirements of an ion gun were written into the specification of a central trajectory, and a gun design meeting these specifications was found Although this particular design is not suitable in all respects for a practical ion gun, the synthesis method is shown to be a powerful tool in obtaining a gun design to meet specific requirements

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